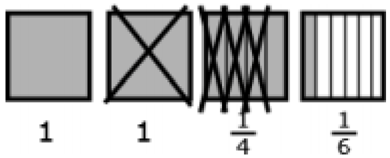
Use equivalent fractions as a strategy to add and subtract fractions (Standards 5.NF.1–2).	
Standard 5.NF.1 Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. *For example, 2/3 + 5/4 = 8/12 + 15/12 = 23/12. (In general,* a/b + c/d = (ad + bc)/bd.*)*	
Concepts and Skills to Master	
• Understand why fractions and mixed numbers must have common denominators to be added or subtracted • Use visual representations to explain the need for common denominators when adding and subtracting fractions and mixed numbers • Use multiple strategies to find common denominators to add or subtract fractions including mixed numbers (See strategies below) • Identify and select efficient strategies to compose and decompose fractions, whole numbers, and mixed numbers flexibly based on the numbers and operations being used in the problem. • Connect visual models to numerical representations Teacher Note: It is not necessary to find a least common denominator to calculate sums of fractions, and in fact the effort of finding a least common denominator is a distraction from understanding algorithms for adding or subtracting fractions. Also, not all fractions need to be expressed in lowest terms. Greatest common factor and least common multiple are introduced in Standard 6.NS.4 and are not needed for an understanding of addition and subtraction of fractions.	
Related Standards: Current Grade Level	Related Standards: Future Grade Level
5.NF.2 Solve real word problems involving addition and subtraction of fractions **5.NBT.7** Add and subtract decimals to hundredths using concrete models or drawings	**6.EE. 7** Solve problems by writing and solving equations of the form x + a = b where variables may be fractions **7.NS.1** Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram **7.NS.3** Solve real-world and mathematical problems involving the four operations with rational numbers. Computations with rational numbers extend the rules for manipulating fractions to complex fractions
Critical Background Knowledge from Previous Grade Levels	
• Explain why fractions are equivalent by using visual fraction models (4.NF.1) • Generate equivalent fractions by creating common denominators or numerators (4.NF.2) • Understand addition and subtraction of fractions as joining and separating parts of the same whole (4 NF 3.a) • Understand a mixed number is a whole number and a fraction that can also be represented as a fraction greater than 1 (4.NF.3.b) • Add and subtract fractions with like denominators including mixed numbers (4.NF.3c)	
Academic Vocabulary	
Common denominator, unlike denominator, like denominator, fraction greater than one, mixed number, numerator, denominator, equivalent fraction, compose, decompose, common multiple	

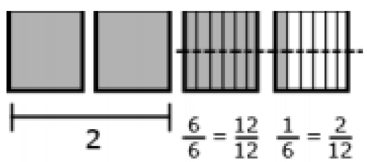
Suggested Models

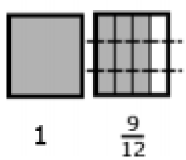
Example: Using an area model to subtract

This model shows $1\frac{3}{4}$ subtracted from $3\frac{1}{6}$ leaving $1 + \frac{1}{4} + \frac{1}{6}$. A student can then convert the fractions to $1 + \frac{3}{12} + \frac{2}{12} = 1\frac{5}{12}$1 + 3/12 + 2/12 = 1 5/12.

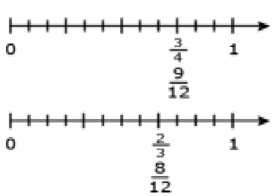


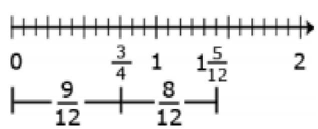
$3\ \frac{1}{6}$ can be expressed as $3\ \frac{2}{12}$.
$3\ \frac{2}{12}$ can be decomposed to create the problem $2\frac{14}{12} - 1\frac{9}{12} = 1\frac{5}{12}$.
This diagram models a way to show how $3\ \frac{1}{6}$ and $1\ \frac{3}{4}$ can be expressed with a denominator of 12 and how $2\frac{14}{12} - 1\frac{9}{12} = 1\frac{5}{12}$ can be solved.





- Linear model





Suggested Strategies

Use visual models including number bonds, number lines, fraction strips, tape diagrams, area models, set models, rulers and equations to do the following:

- Use equivalent fractions as a strategy to find common denominators in order to add and subtract fractions
- Apply understanding of equivalent fractions to rewrite fractions in equivalent forms with common denominators
- Use the Multiplicative Identity Property of 1 to transform a fraction into an equivalent fraction and generate equivalent fractions using this principle (Students may, but need not, use the formal term for this property)
- Find common denominators through common multiples or finding the product of both denominators

Use equivalent fractions as a strategy to add and subtract fractions (Standards 5.NF.1–2).

Standard 5.NF.2 Solve real-world problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators by, *for example, using visual fraction models or equations to represent the problem*. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. *For example, recognize 2/5 + 1/2 = 3/7 as an incorrect result, by observing that 3/7 < 1/2.*

Concepts and Skills to Master

- Understand why fractions and mixed numbers must have common denominators to be added or subtracted
- Use visual representations to explain the need for common denominators when adding and subtracting fractions and mixed numbers
- Use multiple strategies to find common denominators to add or subtract fractions including mixed numbers (See strategies below)
- Identify and select efficient strategies to compose and decompose fractions, whole numbers, and mixed numbers flexibly based on the numbers and operations being used in the problem
- Connect visual models to numerical representations
- Solve real-world problems involving addition and subtraction of fractions, including mixed numbers
- Mentally estimate and assess the reasonableness of an answer

Teacher Note: It is not necessary to find a least common denominator to calculate sums of fractions, and in fact the effort of finding a least common denominator is a distraction from understanding algorithms for adding or subtracting fractions. Also, not all fractions need to be expressed in lowest terms. Greatest common factor and least common multiple are introduced in Standard 6.NS.4 and are not needed for an understanding of addition and subtraction of fractions.

Related Standards: Current Grade Level	Related Standards: Future Grade Level
5.NF.1 Add and subtract fractions with unlike denominators (including mixed numbers) **5.NBT.7** Add and subtract decimals to hundredths using concrete models or drawings	**6.EE. 7** Solve problems by writing and solving equations of the form x + a = b where variables may be fractions **7.NS.1** Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram **7.NS.3** Solve real-world and mathematical problems involving the four operations with rational numbers. Computations with rational numbers extend the rules for manipulating fractions to complex fractions

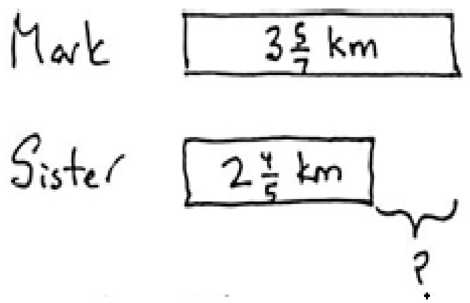
Critical Background Knowledge from Previous Grade Levels

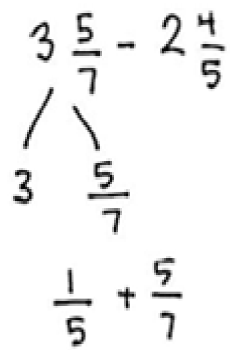
- Explain why fractions are equivalent by using visual fraction models (4.NF.1)
- Generate equivalent fractions by creating common denominators or numerators (4.NF.2)
- Understand addition and subtraction of fractions as joining and separating parts of the same whole (4 NF 3.a)
- Understand a mixed number is a whole number and a fraction that can also be represented as a fraction greater than 1 (4.NF.3.b)
- Add and subtract fractions with like denominators including mixed numbers (4.NF.3c)

Academic Vocabulary

fraction greater than one, mixed number, numerator, denominator, like denominators, unlike denominators, common denominators, equivalent fractions, compose, decompose, common multiple, estimate, reasonableness

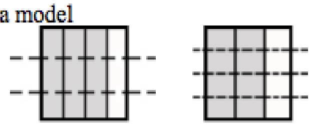
Suggested Models





Example:
Jerry was making two different types of cookies. One recipe needed 3/4 cup of sugar and the other needed 2/3 cup of sugar. How much sugar did he need to make both recipes?

- Mental estimation:
  A student may say that Jerry needs more than 1 cup of sugar but less than 2 cups. An explanation may compare both fractions to ½ and state that both are larger than ½ so the total must be more than 1. In addition, both fractions are slightly less than 1 so the sum cannot be more than 2.
- Area model



$\frac{3}{4}$ cup of sugar $\qquad$ $\frac{2}{3}$ cup of sugar

$\frac{3}{4}=\frac{9}{12}$ $\qquad$ $\frac{2}{3}=\frac{8}{12}$ $\qquad$ $\frac{3}{4}+\frac{2}{3}=\frac{17}{12}=\frac{12}{12}+\frac{5}{12}=1\frac{5}{12}$

Suggested Strategies

Use visual models including number bonds, number lines, fraction strips, tape diagrams, area models, set models, rulers and equations to do the following:

- Use equivalent fractions as a strategy to find common denominators in order to add and subtract fractions
- Apply understanding of equivalent fractions to rewrite fractions in equivalent forms with common denominators
- Use the Multiplicative Identity Property of 1 to transform a fraction into an equivalent fraction and generate equivalent fractions using this principle (Students may, but need not, use the formal term for this property.)
- Find common denominators through common multiples or finding the product of both denominators
- Use benchmark fractions (0, $\frac{1}{2}$, 1) to estimate and assess the reasonableness of an answer

Apply and extend previous understandings of multiplication and division to multiply and divide fractions (Standards 5.NF.3–7).

Standard 5.NF.3 Interpret a fraction as division of the numerator by the denominator (*a/b = a ÷ b*). Solve real-world problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, through the use of visual fraction models or equations to represent the problem. *For example, interpret 3/4 as the result of dividing three by four, noting that 3/4 multiplied by four equals three, and that when three wholes are shared equally among four people each person has a share of size 3/4. If nine people want to share a 50-pound sack of rice equally by weight, how many pounds of rice should each person get? Between what two whole numbers does your answer lie?*

Concepts and Skills to Master

- Understand that a fraction is a way to represent the division of two quantities (a/b = a÷b)
- Rewrite a whole-number division expression as a fraction. Know that 3/5 "three fifths" can also be interpreted as "3 divided by 5"
- Create story contexts to represent problems involving division of whole numbers to include remainders written as fractions

Related Standards: Current Course	Related Standards: Future Courses
5.NF.4 Multiply a fraction or a whole number by a fraction	**6.RP.2** Understand ratio concepts and ratio reasoning to solve problems
5.NF.5 Interpret multiplication as scaling	**6.G.2** Solve volume problems for solids with unit fraction edge lengths
5.NF.7 Divide whole numbers and unit fractions by each other	**7.NS.2** Apply and extend operations with fractions to add, subtract, multiply, and divide irrational numbers

Critical Background Knowledge from Previous Grade Levels

- Understand multiplication of a whole number and a fraction as the concept of repeated addition of unit fractions. (4.NF.4)
- Multiply and divide to solve word problems involving whole numbers. (4.OA.2)
- Divide whole numbers by whole numbers. (3.OA.2)

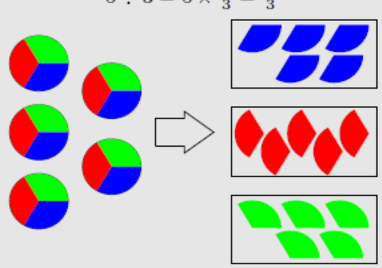
Academic Vocabulary

numerator, denominator, fraction greater than one, mixed number, quotient, divisor, dividend, remainder, fair share, equal shares, sharing, equal size pieces

Suggested Models

How to share 5 objects equally among 3 shares:

$5 \div 3 = 5 \times \frac{1}{3} = \frac{5}{3}$



If you divide 5 objects equally among 3 shares, each of the 5 objects should contribute $\frac{1}{3}$ of itself to each share. Thus each share consists of 5 pieces, each of which is $\frac{1}{3}$ of an object, and so each share is $5 \times \frac{1}{3} = \frac{5}{3}$ of an object.

Suggested Strategies

- Use concrete and visual fraction models and equations to represent a problem
- Convert a division problem into a multiplication problem involving a whole number and unit fraction
- Use whole-number multiplication to find the closest whole-number quotient and then partition the remainder into equal groups
- Use contexts of word problems to evaluate reasonableness of answers and remainders

If nine people want to share a 50-pound sack of rice equally by weight, how many pounds of rice should each person get?

9 x 5 = 45 pounds so each person receives 5 pounds with 5 pounds remaining. Partitioning the remaining 5 pounds give each person $\frac{5}{9}$ pounds per person. So each person gets $5\frac{5}{9}$ pounds of rice.

Image Source: http://commoncoretools.me/wp-content/uploads/2011/08/ccss_progression_nf_35_2013_09_19.pdf

Apply and extend previous understandings of multiplication and division to multiply and divide fractions (Standards 5.NF.3–7).

Standard 5.NF.4 Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction.
a. Interpret the product (*a/b*) x *q* as a parts of a partition of q into b equal parts; equivalently, as the result of a sequence of operations *a* x *q* ÷ *b* using a visual fraction model.
For example, use a fraction model to show (2/3) x 4 = 8/3, and create a story context for this equation. Do the same with (2/3) x (4/5) = 8/15. (In general, (a/b) *x* (c/d) = ac/bd.*)*
b. Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas.

Concepts and Skills to Master

- Understand that a whole number multiplied by a fraction can be represented as repeated addition ($2\ x\ \frac{1}{4} - \frac{1}{4} + \frac{1}{4}$)
- Create a story context for an equation of the form (a/b) x q
- Multiply and represent a fraction by a whole number
- Multiply and represent a fraction by a fraction including fractions greater than 1
- Understand that the area of a rectangle is measured in square units and that square units may be fractional units
- Create area models to illustrate the meaning of multiplying fractions and explain the model's relationship to both factors and the product
- Find the area of a rectangle with fractional side lengths by tiling the area with unit squares
- Find that the area of a rectangle with fractional sides is the same as the product of the side lengths

Related Standards: Current Course	Related Standards: Future Courses
5.NBT.7 Perform operations with multi-digit whole numbers and with decimals to the hundredths **5.NF.5b** Apply and extend previous understandings of multiplication and division to multiply and divide fractions	**6.EE.7** Solve real-world and mathematical problems by writing and solving equations of the form ax=b for cases in which a, b and x are all non-negative rational numbers **6.G.1 - 4** Solve real-world and mathematical problems involving area, surface area and volume **7.NS.2a** Apply and extend previous understandings of multiplication as an extension from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations **7.SP.3** Draw informal comparative inferences about two populations

Critical Background Knowledge from Previous Grade Levels

- Apply and extend previous understandings of multiplication to multiply a fraction by a whole number (4.NF.4)
- A square with side length one unit, called "a unit square," is said to have "one square unit" of area, and can be used to measure area. (3.MD.5a)
- A plane figure which can be covered without gaps or overlaps by n unit squares is said to have an area of n square units. (3.MD.5 b)
- Geometric measurement: Understand concepts of area and relate area to multiplication and to addition. (3.MD.7c)

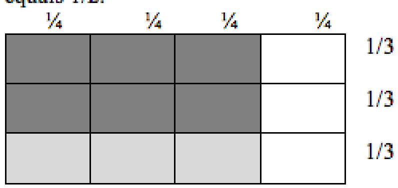
Academic Vocabulary

partition, factor, product, numerator, denominator, fraction, whole number, unit Fraction, equivalent, area, length, width, square unit, array, dimension, tiling

Suggested Models

Student 1

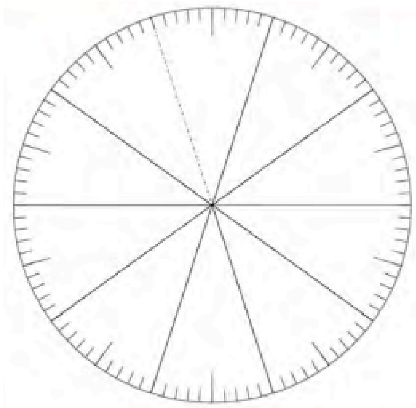
I drew a rectangle to represent the whole class. The four columns represent the fourths of a class. I shaded 3 columns to represent the fraction that are boys. I then split the rectangle with horizontal lines into thirds. The dark area represents the fraction of the boys in the class wearing tennis shoes, which is 6 out of 12. That is 6/12, which equals 1/2.



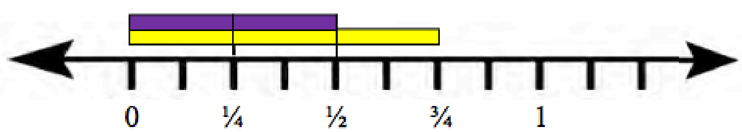
Student 3

Fraction circle could be used to model student thinking.

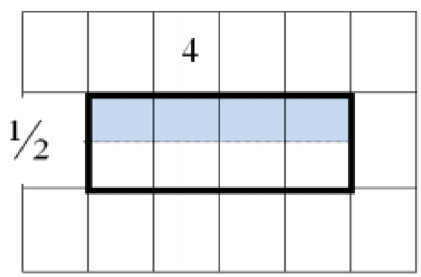
First I shade the fraction circle to show the ¾ and then overlay with 2/3 of that?



Student 2

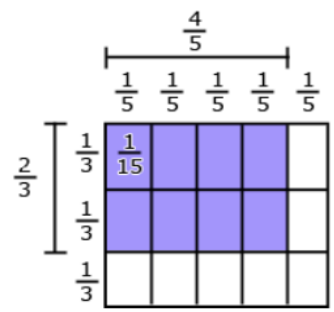


Example: The home builder needs to cover a small storage room floor with carpet. The storage room is 4 meters long and half of a meter wide. How much carpet do you need to cover the floor of the storage room? Use a grid to show your work and explain your answer. In the grid below I shaded the top half of 4 boxes. When I added them together, I added ½ four times, which equals 2. I could also think about this with multiplication ½ x 4 is equal to 4/2 which is equal to 2.



Suggested Strategies

- Create a story context for an equation of the form (a/b) x q (Students create a story problem for 3/5 x 6 such as, • Isabel had 6 feet of wrapping paper. She used 3/5 of the paper to wrap some presents. How much does she have left? • Every day Tim ran 3/5 of mile. How far did he run after 6 days? (Interpreting this as 6 x 3/5)
- Use visual fraction models (area models, tape diagrams, number lines, circles, folding) to multiply a whole number by a fraction
- Create area models to find the area of a rectangle with fractional side lengths and explain the model's relationship to both factors and the product
- Create area models Use graph paper/dot paper to make pictorial representations of area
- Write equations to represent area models showing a product of fractions



The area model and the line segments show that the area is the same quantity as the product of the side lengths.

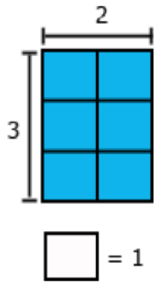
Image Sources: http://commoncoretools.me/wp-content/uploads/2011/08/ccss_progression_nf_35_2013_09_19.pdf; http://www.dpi.state.nc.us/docs/curriculum/mathematics/scos/5.pdf

Apply and extend previous understandings of multiplication and division to multiply and divide fractions (Standards 5.NF.3–7).	
Standard 5.NF.5 Interpret multiplication as scaling. **a.** Compare the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication. *For example, the products of expressions such as 5 x 3 or ½ x 3 can be interpreted in terms of a quantity, three, and a scaling factor, five or ½. Thus in addition to knowing that 5 x 3 = 15, they can also say that 5 x 3 is five times as big as three, without evaluating the product. Likewise they see ½ x 3 as half the size of three.* **b.** Explain why multiplying a given number by a fraction greater than one results in a product greater than the given number (recognizing multiplication by whole numbers greater than one as a familiar case); explain why multiplying a given number by a fraction less than one results in a product smaller than the given number; and relate the principle of fraction equivalence. *For example, 6/10 = (2x3)/(2x5). In general,* a/b = *(*n *x* a*)/(*n *x* b*) has the effect of multiplying* a/b *by one*.	
Concepts and Skills to Master	
• Understand relationships between the size of factors and products • Use estimation to check the reasonableness of the products • Understand multiplication as scaling as expressions that can be interpreted in terms of quantity and scaling factor (5 x 3 is 5 times as big as 3. $\frac{1}{2}$ x 3 is half the size of 3) • Explain why multiplying a given number by a fraction greater than one results in a product greater than the given number • Explain why multiplying a given number by a fraction less than one results in a product smaller than the given number • Understand fraction equivalence	
Related Standards: Current Grade Level	Related Standards: Future Grade Levels
5.OA.2 Write and interpret numerical expressions **5.NF.4b** Find the area of a rectangle with fractional side lengths	**6.RP.1** Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities **6.RP.2** Understand the concept of a unit rate a/b associated with a ratio a:b with b not equal to 0, and use rate language in the context of a ratio relationship **6. RP.3** Use ratio and rate reasoning to solve real-world and mathematical problems.
Critical Background Knowledge from Previous Grade Levels	
• Use the four operations to solve word problems. (4.MD.2) • Explain why a fraction a/b is equivalent to a fraction (n x a)/(n x b) by using visual fraction models (4.NF.1) • Compare two fractions with different numerators and different denominators (4.OA.2) • Interpret a multiplication equation as a comparison (4.OA.1) • Interpret products of whole numbers (3.OA.1) • Interpret whole-number quotients of whole numbers (3.OA.2)	
Academic Vocabulary	
scaling, array, factor, product, x means “of”, compare, increase, decrease, fraction greater than 1, fraction less than 1, mixed number	

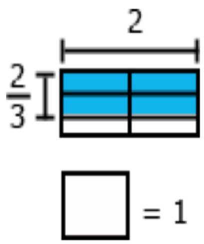
Suggested Models	Suggested Strategies
See below	See below

Suggested Models

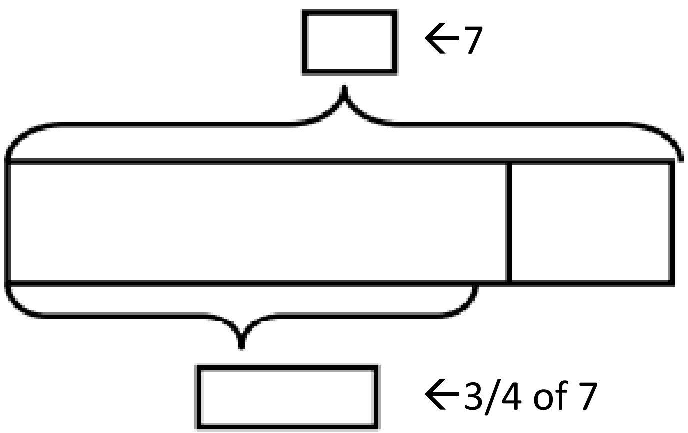
- Rectangle with dimensions of 2 and 3 showing that 2 x 3 = 6.



- Rectangle with dimensions of 2 and $\frac{2}{3}$ showing that 2 x 2/3 = 4/3



Example: $\frac{3}{4}\ x$ 7 is less than 7 because 7 is multiplied by a factor less than 1 so the product must be less than 7.



Suggested Strategies

- Draw models to compare, and reason, about the size of products in relationships to the size of various factors.
- Use area models to demonstrate the concept of scaling
- Construct viable arguments and critique the reasoning of others about the size of a product compared to the size of one factor on the basis of the size of the other factor.
- Use models and/or words to explain why multiplying a given number by a fraction greater than one results in a product greater than the given number
- Use models and/or words to explain why multiplying a given number by a fraction less than one results in a product smaller than the given number
- Work with multiplying by unit fractions

Image Source: http://www.dpi.state.nc.us/docs/curriculum/mathematics/scos/5.pdf

Apply and extend previous understandings of multiplication and division to multiply and divide fractions (Standards 5.NF.3–7).

Standard 5.NF.6 Solve real-world problems involving multiplication of fractions and mixed numbers, f*or example, by using visual fraction models or equations to represent the problem*.

Concepts and Skills to Master

- Understand and use various strategies to interpret word problems involving multiplication of fractions and mixed numbers (fraction by a fraction, fraction by a mixed number, mixed number by mixed number)
- Write an equation to represent a word problem and solve the equation

Related Standards: Current Grade Level	Related Standards: Future Grade Levels
5.NF.4 Apply and extend previous understandings of multiplication to multiply a fraction or whole number by an fraction. **5.NF.5** Interpret multiplication as scaling **5.MD.2** Make a line plot to display a data set or measurements in fractions and multiply fractions to solve problems	**6.EE.7** Solve real-world and mathematical problems by writing and solving equations of the form ax = b for cases in which a, b and x are all nonnegative rational numbers **6.RP.3** Use ratio and rate reasoning to solve real-world problems **6.G.1 - 4** Solve real-world and mathematical problems involving area, surface area and volume **7.NS.2a** Apply and extend previous understandings of multiplication as an extension from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations

Critical Background Knowledge from Previous Grade Levels

- Apply and extend previous understandings of multiplication to multiply a fraction by a whole number (4.NF.4)
- Use the four operations to solve word problems involving simple fractions (4.MD.2)
- Interpret a multiplication equation as a comparison; Multiply to divide to solve word problems involving multiplicative comparison (4.0A.1, 4.0A.2)
- Interpret products of whole numbers (3.0A.1)

Academic Vocabulary

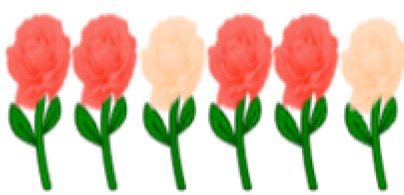
Equation, factors, products, fraction, mixed number

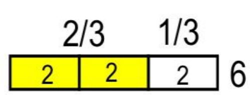
Suggested Models

Example: Evan bought 6 roses for his mother. of them were red. How many red roses were there? Using a visual, a student divides the 6 roses into 3 groups and counts how many are in 2 of the 3 groups.

$\frac{2}{3} x\ 6 = \frac{12}{3} = 4$

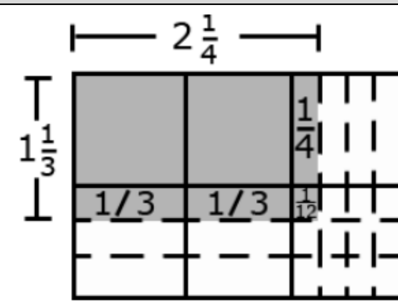
4 roses are red





Example: Mary and Joe determined that the dimensions of their school flag needed to be $1\frac{1}{3}$ ft. by $2\frac{1}{4}$ ft. What will be the area of the school flag?

A student can draw an array to find this product and can also use his or her understanding of decomposing numbers to explain the multiplication.



Suggested Strategies

- Use concrete and pictorial area models to represent and make sense of real world problems (unit bars, number lines, area models, linear models, pattern blocks, fraction circles)

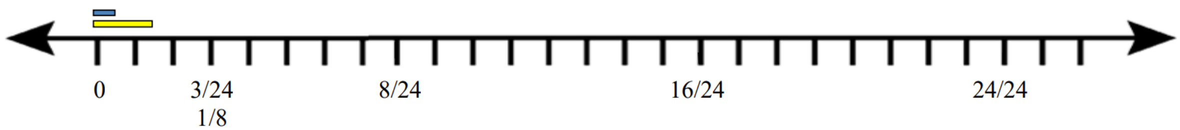
Image Source: http://www.dpi.state.nc.us/docs/curriculum/mathematics/scos/5.pdf

Apply and extend previous understandings of multiplication and division to multiply and divide fractions (Standards 5.NF.3–7).	
Standard 5.NF.7 Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions. Use strategies to divide fractions by reasoning about the relationship between multiplication and division. Division of a fraction by a fraction is not a requirement at this grade. **a.** Interpret division of a unit fraction by a non-zero whole number, and compute such quotients. *For example, create a story context for (1/3) ÷ 4, and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that (1/3) ÷ 4 = 1/12 because (1/12) x 4 = 1/3.* **b.** Interpret division of a whole number by a unit fraction, and compute such quotients. *For example, create a story context for 4 ÷ (1/5), and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that 4 ÷ (1/5) = 20 because 20 x (1/5) = 4.* **c.** Solve real-world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions, *for example, by using visual fraction models and equations to represent the problem*. *For example, how much chocolate will each person get if three people share 1/2 lb. of chocolate equally? How many 1/3-cup servings are in two cups of raisins?*	
Concepts and Skills to Master	
• Understand and use visual models to divide a unit fraction by an non-zero whole number (e.g., ⅓ *÷ 4)* • Understand and use visual models to divide a whole number by a unit fraction. (e.g., *4 ÷ ⅕)* • Solve real word problems using division of fractions. • Understand and use the inverse relationship between multiplication and division to reason and solve real world problems. Teacher Note: This standard is limited to dividing with whole numbers and unit fractions. Fractions divided by fractions will be introduced in 6th grade. This standard should be taught with context and visual models.	
Related Standards: Current Grade Level	Related Standards: Future Grade Levels
5.NF.4 Apply and extend previous understanding of multiplication to multiply a fraction or whole number by a fraction. **5.NF.6** Solve real word-world problems involving multiplication of fractions and mixed numbers. **5.NBT.7** Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction.	**6.NS.1** Interpret and compute quotients of fractions by fractions by applying visual fraction models, equations, and the relationship between multiplication and division. Solve real world problems and explain the meaning of quotients in fraction division problems. **6.RP.2** Understand the concept of a unit rate a/b associated with a ratio a:b with b ? 0, and use rate language in the context of a ratio relationship.
Critical Background Knowledge from Previous Grade Levels	
• Apply and extend previous understanding of multiplication to multiply a fraction by a whole number (4.NF.4) • Explain why fraction are equivalent by using visual fraction models (4.NF.1) • Generate equivalent fraction by creating common denominators or numerators (4.NF.2) • Understand properties of multiplication and the relationship between multiplication and division (3.0A.6) • Understand that a unit fraction has a numerator of one and a non-zero denominator. (3.NF.1)	
Academic Vocabulary	
Unit fraction, whole number, quotient, dividend, divisor, equation, inverse operations	

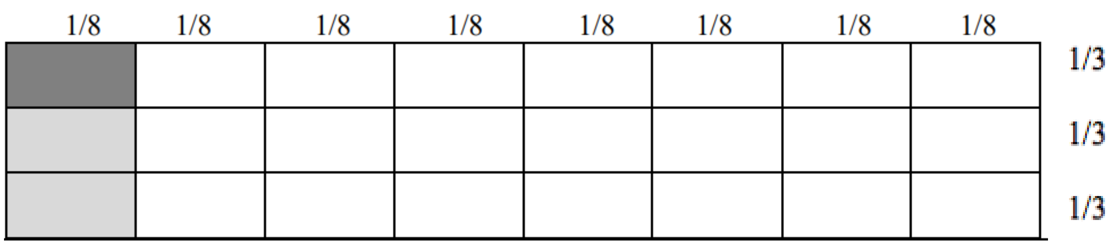
Suggested Models

Example: You have 1/8 of a bag of pens and you need to share them among 3 people. How much of the bag does each person get?

Expression 1/ 8 ÷ 3



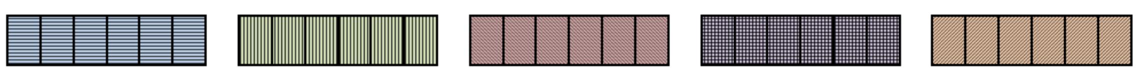
I drew a rectangle and divided it into 8 columns to represent my 1/8. I shaded the first column. I then needed to divide the shaded region into 3 parts to represent sharing among 3 people. I shaded one-third of the first column even darker. The dark shade is 1/24 of the grid or 1/24 of the bag of pens.



Example: Create a story context for 5 ÷ 1/6. Find your answer and then draw a picture to prove your answer and use multiplication to reason about whether your answer makes sense. How many 1/6 are there in 5?

The bowl holds 5 Liters of water. If we use a scoop that holds 1/6 of a Liter, how many scoops will we need in order to fill the entire bowl?

I created 5 boxes. Each box represents 1 Liter of water. I then divided each box into sixths to represent the size of the scoop. My answer is the number of small boxes, which is 30. That makes sense since 6 x 5 = 30.



1 = 1/6 + 1/6 + 1/6 + 1/6 + 1/6 a whole has 6/6 so five wholes would be 6/6 + 6/6 + 6/6 + 6/6 + 6/6 =30/6

Suggested Strategies

- Use various visual fraction models to illustrate division of a unit fraction by a non-zero whole number and division of a whole number by a unit fraction
- Create story context to illustrate division of a unit fraction by a non-zero whole number and division of a whole number by a unit fraction
- Solve real-world problems using manipulatives
- Give students an equation and have them come up with a real world problem that represents the equation
- Relate multiplication of fractions to division of fractions

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TABLE 2. Common multiplication and division situations.[1]

	Unknown Product	Group Size Unknown *("How many in each group?" Division)*	Number of Groups Unknown *("How many groups?" Division)*
	3 × 6 = ?	**3 × ? = 18 and 18 ÷ 3 = ?**	**? × 6 = 18 and 18 ÷ 6 = ?**
EQUAL GROUPS	There are 3 bags with 6 plums in each bag. How many plums are there in all? ***Measurement example.*** You need 3 lengths of string, each 6 inches long. How much string will you need altogether?	If 18 plums are shared equally into 3 bags, then how many plums will be in each bag? ***Measurement example.*** You have 18 inches of string, which you will cut into 3 equal pieces. How long will each piece of string be?	If 18 plums are to be packed 6 to a bag, then how many bags are needed? ***Measurement example.*** You have 18 inches of string, which you will cut into pieces that are 6 inches long. How many pieces of string will you have?
ARRAYS[2]	There are 3 rows of apples with 6 apples in each row. How many apples are there?	If 18 apples are arranged into 3 equal rows, how many apples will be in each row?	If 18 apples are arranged into equal rows of 6 apples, how many rows will there be?
AREA[3]	What is the area of a 3 cm by 6 cm rectangle?	A rectangle has area 18 square centimeters. If one side is 3 cm long, how long is a side next to it?	A rectangle has area 18 square centimeters. If one side is 6 cm long, how long is a side next to it?
COMPARE[4]	A blue hat costs $6. A red hat costs 3 times as much as the blue hat. How much does the red hat cost? ***Measurement example.*** A rubber band is 6 cm long. How long will the rubber band be when it is stretched to be 3 times as long?	A red hat costs $18 and that is 3 times as much as a blue hat costs. How much does a blue hat cost? ***Measurement example.*** A rubber band is stretched to be 18 cm long and that is 3 times as long as it was at first. How long was the rubber band at first?	A red hat costs $18 and a blue hat costs $6. How many times as much does the red hat cost as the blue hat? ***Measurement example.*** A rubber band was 6 cm long at first. Now it is stretched to be 18 cm long. How many times as long is the rubber band now as it was at first?
GENERAL	$a \times b = ?$	$a \times ? = p$ and $p \div a = ?$	$? \times b = p$ and $p \div b = ?$

[1] The first examples in each cell are examples of discrete things. These are easier for students and should be given before the measurement examples.

[2] The language in the array examples shows the easiest form of array problems. A harder form is to use the terms rows and columns: The apples in the grocery window are in 3 rows and 6 columns. How many apples are in there? Both forms are valuable.

[3] Area involves arrays of squares that have been pushed together so that there are no gaps or overlaps, so array problems include these especially important measurement situations.

[4] Multiplicative Compare problems appear first in Grade 4, with whole-number values in all places, and with the "times as much" language in the table. In Grade 5, unit fractions language such as "one third as much" may be used. Multiplying and unit fraction language change the subject of the comparing sentence, e.g., "A red hat costs A times as much as the blue hat" results in the same comparison as "A blue hat costs 1/A times as much as the red hat," but has a different subject.